A METHOD OF SELECTING PORTS OF ENTRY FOR A DEVELOPING COUNTRY

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THESIS

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by

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ABSTRACT

A solution algorithm is presented which solves the problem of selecting ports of entry for the case where the number of ports of entry is constrained to be small relative to the total number of ports. The algorithm initially considers all ports in the system considered as candidates for ports of entry and proceeds by eliminating from further consideration one port at a time until the required number is attained. An attempt is made to remain as close as possible to the unconstrained solution by eliminating at each iteration the port that has the least effect on the objective function value. It is pointed out that the algorithm can yield non-optimal results in some cases but the solution is still better than most feasible solutions.

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I. INTRODUCTION

This thesis deals with a particular combinatorial problem that can be faced by any developing country which relies extensively on its maritime fleet as a principal means of transporting goods between its different regions. Specifically, the problem addressed is that of selecting ports of entry.

Suppose a country P normally routes imported goods through a predetermined subset of its ports, henceforth referred to as ports of entry, for the purpose of complying with customs requirements imposed by the country. Country P presently has a few ports of entry which have recently become inadequate to handle the rapid increase of the demand for imported goods. Conceivably, country P can develop all of its ports to receive imported goods so that all goods destined for a certain region can be sent directly to the nearest port, thereby eliminating the added cost of transshipment. However, ports of entry are much more expensive to operate and maintain than the ordinary ports that handle only coastwise trade. Furthermore, the initial cost of port development might be high. As a consequence, the development and maintenance of all ports may be well beyond the resources available. Hence, country P would like to increase its ports of entry to an acceptable number which can relieve the pressure on its present ports of entry, effectively serve the needs of the country and, at the same time, stay within a special budget appropriated for ports development.

The problem faced by country P is that of determining new ports of entry. Since all goods imported by country P have to go through a port

of entry, the selection of the new ports of entry must consider the location of the existing ones so that the final set of ports of entry can effectively serve the remaining ones. A measure that can be used in determining the effectiveness of the system for a given choice of ports of entry is the total cost incurred in satisfying the total demand for imported goods. This total cost consists of the cost of operating and maintaining ports of entry and the cost of transshipping imported goods to their destination. The problem then reduces to the selection of ports of entry so that the total cost incurred in satisfying the total demand for imported goods is minimized.

Section II presents a rigorous formulation of the problem as a transshipment problem with a constraint on the number of sources used. For convenience and simplification, the cost of developing a port into a port of entry is initially assumed constant and the same for any port selected. It is also assumed that the special budget for ports development is a linear multiple of the constant cost of developing a port. This facilitates the determination of the number of ports of entry that can be developed for the country. The formulation of the problem is modified slightly to allow for a solution using some existing network flow algorithms. Section III proposes an algorithm that can be used to solve the problem. The algorithm utilizes the Out of Kilter Algorithm of Ford and Fulkerson (Ref. 1) to determine the optimal flows, ignoring the ports development budget restriction. An example is also presented and explained. Section IV discusses the efficiency of the algorithm and the merits of the results obtained. An extension brought about by the relaxation of some of the assumptions is also presented and discussed. Section V summarizes the report and recommends some areas for further investigation.

II. PROBLEM FORMULATION

Formally, the problem is to select some ports out of all possible ports through which importations can be made so that the following conditions are satisfied;

1. The ports selected must include the existing ports of entry. In this thesis the number of ports selected has to be greater than, or equal to, the present number of ports of entry.

2. The demand for imported goods at each port is satisfied fully. This demand shall be taken as the total of all demands for imported goods of the region that is being served by the port. In order to justify this pooling of demands, it is assumed that all goods destined for any place in the country will pass through the port which serves that place.

3. The quantity of goods imported through a port of entry can not exceed the capacity of the port. This capacity may be difficult to determine due to the stochastic nature of shipment arrivals and the arrival of claims for goods. This report does not deal with this difficulty but rather assumes that a concrete value for port capacity is available.

4. The total cost of satisfying the demands at all ports is minimized. This cost includes the total operating and maintenance cost of ports of entry and the total cost of transshipping the imported goods demanded at ordinary ports. It is assumed initially that cost is a linear multiple of the quantity of goods entered at or transshipped from ports of entry.

5. The total cost of developing the ports selected can not exceed the special budget appropriated for ports development.

A. DEFINITIONS AND ASSUMPTIONS

Throughout this report, imported goods referred to are only those that fall under the category of general cargo. It will be assumed that handling charges and transshipment costs per unit of shipping is the same for all goods in this category. A general cargo can therefore be considered as the single commodity that is entered at and shipped from ports of entry.

The context of any definition in this section is a graph G(N,A) composed of a non-empty set of nodes N and a set of directed arcs A that link the elements of N. A node is denoted by an integer. The ordered pair (i,j) denotes a directed arc incident from node i and incident to node j.

Associated with each arc (i,j) are constants c_{ij}, the cost incurred by a unit of flow across the arc; L_{ij}, a lower bound on the amount of flow that is sent across the arc; and M_{ij}, the flow upper bound or arc capacity. Let X_{ij} denote the actual flow sent across arc (i,j).

The following definitions are made in connection with the formulation of the problem and the development of a solution algorithm;

N = the set of all ports of the country.

A = The set of all legs of all commercial shipping routes linking the ports of the country.

Node o = the common single source node. This can be thought of as the international market which is the source of all goods imported by the country.

Node n = the common single sink node. This represents the importers as a group who receive all goods imported.

S = the set of all source arcs, i.e., those arcs that are incident from node o.

T = the set of all demand arcs, i.e., those arcs that are incident to node n.

N' = the set of all nodes in an expanded graph including the single source and single sink nodes.

A' = the set of all arcs of an expanded graph including the sets of source and demand arcs.

D = the special budget for ports development.

d_{oj} = the cost of developing part j into a port of entry.

c_{oj} = the increase in operating and maintenance cost per unit of import shipping through port j after it is made a port of entry.

c_{ij} = the cost per unit of shipping from port i to port j.

c_{in} = the artificial cost for the demand at port i (assumed zero).

L_{oj} = the minimum quantity of goods entered at port j if it is a port of entry.

L_{ij} = a lower bound on the quantity of goods that can be shipped from port i to port j (assumed zero).

L_{in} = a lower bound on the quantity demanded at port i.

M_{oj} = the maximum import capacity of port j.

M_{ij} = the maximum shipping capacity for the leg from port i to port j for $i \neq o$ and $j \neq n$.

M_{in} = the maximum quantity demand at port i.

K = the set of all present ports of entry.

J_s = $\{j : X_{oj} > 0, (o,j) \in S\}$, the set of ports which are candidates for the final choice of ports of entry at iteration s.

$\#(J_s)$ = the number of elements of the set J_s.

$\overline{K},\overline{J}_s$ = the complements of the sets K,J.

r = the maximum number of ports of entry desired for the system.

X_{oj} = the actual amount of goods imported through port j (or the amount of flow across arc (o,j)).

X_{ij} = the actual amount of goods shipped through the leg from port i to port j.

X_{in} = the actual amount of the demand satisfied for port i.

Y_{oj} = the amount of flow allocated to the minimum cost route(s) found so far by the subroutine.

$R_{oj}^{(m)}$ = the least cost route from node o to node j at the end of iteration m.

$a_{oj}^{(m)}$ = the total cost increase per unit of shipping when flow across arc (o,j) is recirculated through route $R_{oj}^{(m)}$.

$W_{oj}^{(m)}$ = the necessary amount of flow that should be sent through the route $R_{oj}^{(m)}$.

C_j = the minimum total cost of recirculating the flow across arc (o,j).

The following assumptions are initially made in the formulation of the problem and the solution algorithm presented;

1. Any transshipment requirement in satisfying the total demand can be accommodated by the existing maritime fleet of the country.

2. All goods destined for a port will be shipped via the cheapest route.

3. The demands at ports of entry will be satisfied by goods directly imported.

4. The capacity of a port is not less than the demand for imported goods of the region that it serves.

5. All costs are linear multiples of the quantity of goods imported and/or transshipped.

6. The development cost is the same for any port selected.

7. The special budget for ports development is a linear multiple of the constant cost of developing any port selected.

An extension brought about by the relaxation of some of the above assumptions will be discussed in Section IV.

B. MATHEMATICAL FORMULATION

The sets of ports N of the country together with the set of commercial shipping routes A that link these ports can be represented by a graph G(N,A). Associated with node $j \in N$ is a capacity M_{oj}, a demand for imported goods M_{in} and a cost c_{oj}. Associated with each arc $(i,j) \in A$ are the bounds $L_{ij} = 0$, $M_{ij} = \infty$, and the cost per unit of shipping, c_{ij}.

Since the required set of ports of entry are yet to be determined, initially consider all the ports $j \in N$ as if these are ports of entry. The initial graph G(N,A) would then be of the form of the graph in Figure 1 of Appendix A.

The graph can be expanded to G(N´,A´) by the addition of a common source node o, a common sink node n, and the sets of arcs

$$S = \{(o,j):j=1,..,n-1\} \text{ and } T = \{(i,n):i=1,...,n-1\}$$

linking the original network with the new source and sink. The resulting graph G(N´,A´) will then be of the form of the graph in Figure 2.

With each arc $(o,j) \in S$, associate a lower bound $L_{oj} = M_{jn}$ if $j \in K$ or $L_{oj} = 0$ if $j \in \overline{K}$, an upper bound M_{oj} and a cost c_{oj}. With each arc

$(i,n) \in T$, associate the lower bound $L_{in} = 0$, an upper bound M_{in} and a cost $c_{in} = 0$.

If the parameters associated with the arcs of $G(N',A')$ are interpreted as defined in Section IIA, the network obtained when flow is allowed through the arcs of $G(N',A')$ can appropriately model the import system of the country.

Let $Q = \sum_{i=1}^{n-1} M_{in}$. Then Q represents the maximum total demand for the commodity. The problem, as stated, can be solved using the network as a model by establishing a minimum cost flow pattern for Q throughout the network which satisfies flow conservation at nodes and which uses only as many arcs as the number of ports of entry required.

The number of ports of entry required is obtained indirectly from condition 5 on page 6 which can be stated as

$$\sum_{(o,j)\in S} d_{oj} Z_{oj} \leq D$$

where d_{oj} and D are as defined in Section IIA and

$$Z_{oj} = \begin{cases} 1 & \text{if port } j \text{ is a port of entry.} \\ 0 & \text{otherwise.} \end{cases}$$

With assumptions 6 and 7 on page 10 , the constraint reduces to

$$\sum_{(o,j)\in S} dZ_{oj} \leq dr \quad \text{or} \quad \sum_{(o,j)\in S} Z_{oj} \leq r$$

indicating that the number of ports of entry can not exceed r.

The minimum cost flow allocation of Q through the network where the number of source arcs used is confined to be less than or equal to r can then be formulated into the following mixed integer programming problem PI:

$$
\begin{array}{lll}
& \text{Minimize} \quad \sum_{(o,j)\varepsilon S} c_{oj}X_{oj} + \sum_{(i,j)\varepsilon A} c_{ij}X_{ij} + \sum_{(i,n)\varepsilon T} c_{in}X_{in} & \\
& \text{Subject to} \quad \sum_{(o,j)\varepsilon S} X_{oj} \qquad = Q & \\
& \sum_{(i,k)\varepsilon A'} X_{ik} - \sum_{(j,i)\varepsilon A'} X_{ji} \qquad = 0 & \forall i=1,\ldots,n-1 \\
\text{(PI)} & \sum_{(i,n)\varepsilon T} X_{in} \qquad = -Q & \\
& \sum_{(o,j)\varepsilon S} Z_{oj} \qquad \leq r & \\
& L_{ij} \leq X_{ij} \leq M_{ij} & \forall (i,j)\varepsilon A' \\
& X_{oj} \leq Z_{oj}M_{oj} & \forall (o,j)\varepsilon S \\
& 0 \leq Z_{oj} \leq 1 & Z_{oj} \text{ integer}
\end{array}
$$

The solution to PI, aside from determining optimal flow through the network, will also have r or fewer arcs $(o,j)\varepsilon S$ with $X_{oj} > 0$. The ports corresponding to the nodes j to which these arcs are incident to will then represent the optimal choice of ports of entry.

Consider the problem PI without the constraint on the number of source arcs $(o,j)\varepsilon S$ used. This problem reduces to the following problem PII:

$$
\begin{array}{lll}
& \text{Minimize} \quad \sum_{(o,j)\varepsilon S} c_{oj}X_{oj} + \sum_{(i,j)\varepsilon A} c_{ij}X_{ij} + \sum_{(i,n)\varepsilon T} c_{in}X_{in} & \\
& \text{Subject to} \quad \sum_{(o,j)\varepsilon S} X_{oj} \qquad = Q & \\
\text{P(II)} & \sum_{(i,k)\varepsilon A'} X_{ik} - \sum_{(j,i)\varepsilon A'} X_{ji} \qquad = 0 & \forall\ i=1,\ldots,n-1 \\
& \sum_{(i,n)\varepsilon T} X_{in} \qquad = -Q & \\
& L_{ij} \leq X_{ij} \leq M_{ij} & \forall (i,j)\varepsilon A' .
\end{array}
$$

Problem PII is nothing but a transshipment problem that can be solved quite easily using the Out of Kilter Algorithm of Ford and Fulkerson (Ref. 1).

The optimal solution to PII can conceivably satisfy the additional constraint, that is, the number of arcs $(o,j)\varepsilon S$ with $X_{oj} > 0$ does not exceed r. If this is the case, then PI is also solved and the ports corresponding to the nodes j such that $X_{oj} > 0$ are the optimal choices for ports of entry. If, on the other hand, the number of arcs $(o,j)\varepsilon S$ in the optimal solution with $X_{oj} > 0$ is greater than r, then the solution for PII is not feasible for PI.

Consider an optimal solution to PII with a set $J = \{j:X_{oj} > o,(o,j)\varepsilon S\}$ and $\#(J) > r$. If the flow across any $\#(J) - r$ arcs $(o,j)\varepsilon S$ are recirculated so that only r arcs $(o,j)\varepsilon S$ remain with $X_{oj} > 0$, then the resulting solution will be feasible for PI. This property will be the basis for the solution algorithm that is presented in the next section.

III. SOLUTION ALGORITHM

The solution algorithm presented in this section attempts to solve PI by first solving PII and deriving from it a feasible solution to PI. It utilizes the Out of Kilter Algorithm (Ref. 1) and an appropriate minimum route algorithm (see Dreyfus (Ref. 2)) in its development.

The solution algorithm consists of a main algorithm and a subroutine. The main algorithm initially solves PII to obtain the minimum cost flow allocation of Q through the network. This solution represents the optimal unconstrained solution for the import system modeled by the network. It selects the number and location of ports of entry in order to attain the least possible cost incurred in satisfying the demand for imported goods. The set $J_1 = \{j: X_{oj} > 0, (o,j) \varepsilon S\}$ represents the ports selected. From this initial solution, the main algorithm obtains a feasible solution to PI by reducing the number of elements of the set J_1 to r if this is required. This is done by recirculating the flow across $\#(J_1) - r$ arcs in J_1, one at a time. The subroutine selects the arc $(o,j) \varepsilon J_1$ to be eliminated at each iteration by the main algorithm by computing the cost of recirculating the flow across each element of the set J_s and comparing these costs. The arc that yields the least recirculation cost is the one selected by the subroutine at that iteration. The algorithm terminates when the number of elements of the set J_s is reduced to r.

A. THE ALGORITHM

Before starting the algorithm, construct the network $G(N',A')$ for the system under consideration as described in Section IIB.

1. Use the Out of Kilter Algorithm to determine the minimum cost flow allocation of $Q = \sum_{i=1}^{n-1} M_{in}$ through the network.

2. Eliminate all arcs (o,j) for which $X_{oj} = 0$. Set $L_{in} = M_{in}$ for all $i=1,\ldots,n-1$. Set $s=1$.

3. Determine the set

$$J_s = \{j: X_{oj} > 0, (o,j)\varepsilon S\}$$

for the resulting network.

4. If,

a. $\#(J_s) \leq r$, terminate. The solution is optimal and the ports corresponding to the elements of the set J_s are chosen as ports of entry.

b. $\#(J_s) > r$, use the Least Recirculation Cost Subroutine to determine which arc (o,j*) can be eliminated.

5. Recirculate the flow across the arc (o,j*) by increasing the flow across all arcs $(k,l)\varepsilon\ R^{(m)}_{oj*}$ by $W^{(m)}_{oj*}$ for all $m=1,2,\ldots$. Eliminate arc (o,j*) and go back to step 3.

Least Recirculation Cost Subroutine

Modify the existing network from the main algorithm by adding in dummy arcs (i′,j′) for each arc (j,i) with $X_{ji} > 0$ for all $i,j=1,\ldots,n-1$. Set $L_{i'j'} = X_{i'j'} = 0$, $M_{i'j'} = X_{ji}$ and $c_{i'j'} = -c_{ji}$. Delete all arcs (i,n) and arc (n,o).

1. For each arc $(o,j)\varepsilon S$ with $j\varepsilon J_s \cap \overline{K}$, obtain the total recirculation cost C_j through the following steps:

a. Set $C_j = 0$, $Y_{oj} = 0$ and $m=1$.

b. From the resulting network, determine the least cost route $R^{(m)}_{oj}$ from node o to node j which does not contain the arc (o,j) using an

appropriate minimum route algorithm. Compute $a_{oj}^{(m)}$, the cost to recirculate one unit over the route where

$$a_{oj}^{(m)} = \sum_{(k,l)\varepsilon R_{oj}^{(m)}} C_{kl} - C_{oj}.$$

c. If $\underset{(k,l)\varepsilon R_{oj}^{(m)}}{\text{Minimum}} (M_{kl} - X_{kl}) \geq (X_{oj} - Y_{oj})$, let

$$W_{oj}^{(m)} = X_{oj} - Y_{oj}$$

$$C_j = \text{old } C_j + a_{oj}^{(m)} W_{oj}^{(m)}$$

and select another arc (o,j). If none remain, go to step 2.

If $\underset{(k,l)\varepsilon R_{oj}^{(m)}}{\text{Minimum}} (M_{kl} - X_{kl}) < (X_{oj} - Y_{oj})$,

(1) Let $W_{oj}^{(m)} = \underset{(k,l)\varepsilon R_{oj}^{(m)}}{\text{Minimum}} (M_{kl} - X_{kl})$

$$Y_{oj} = \text{old } Y_{oj} + W_{oj}$$

$$C_j = \text{old } C_j + a_{oj}^{(m)} W_{oj}^{(m)}$$

(2) Let $X_{kl} = \text{old } X_{kl} + W_{oj}^{(m)}$ for all arcs $(k,l)\varepsilon R_{oj}^{(m)}$.

(3) Eliminate arc (p,q) for which

$$M_{pq} - X_{pq} = W_{oj}^{(m)}.$$

(4) Increase m to m+1 and return to step 1b.

2. For values of C_j obtained above, determine the arc (o,j^*) for which $C_{j^*} = \underset{j \varepsilon J_s \cap \overline{K}}{\text{Minimum}} C_j$. This chooses the arc (o,j^*) as that arc which can be eliminated with the least increase in flow recirculation cost. Go to step 5 of the main algorithm.

B. AN EXAMPLE

Consider a hypothetical problem where it is desired to increase the number of ports of entry from one to three for a system of seven ports. Let the graphical representation $G(N,A)$ of the system be the graph in Figure 1 of Appendix A. Let the parameter values be those outlined in Table 1 of Appendix B. The problems PI and PII for the system are formulated in Appendix C.

The initial network used to start the algorithm is that in Figure 2 with $K = \{1\}$. To insure that the current port of entry will be included among the $r=3$ ports selected, the lower bound for the source arc $(o,1)$ is set equal to the demand at node 1.

Step 1 of the algorithm requires a solution to PII for a total demand $Q = \sum_{i=1}^{7} M_{i8} = 160$. The solution is shown in Figure 2 with

$$J_1 = \{1,2,4,6,7\} \text{ and } \#(J_1) = 5.$$

The solution implies that the least total cost in satisfying the demand $Q = 160$ can be achieved when the ports corresponding to the elements of J_1 are made ports of entry. However, this is more than the desired number. If only the elements of a proper subset of J_1 are selected, an increase in total cost will be realized. Therefore, the subset of J_1 must be selected in such a way that the increase in total cost is minimized. This is guaranteed for each step by the use of the subroutine.

The results of the subroutine for the first iteration are summarized in Table 2 of Appendix B. Arc (o,6) is selected to leave at this iteration (C_6 = 20).

The results of step 5 of the main algorithm is a reduced network used to start the next iteration. This is shown in Figure 3 with

$$J_2 = \{1,2,4,7\} \text{ and } \#(J_2) = 4.$$

Upon application of the subroutine for the second iteration, arc (o,2) is selected to leave as shown by the summary of the results in Table 3 of Appendix B. Recirculating the flow across the arc (o,2) results in the network in Figure 4. The algorithm is then terminated since $J_3 = \{1,4,7\}$ and $\#(J_3) = 3 = r$.

The resulting solution obtained by the algorithm can easily be shown to be feasible for PI by checking if the additional constraints are satisfied. For the solution, $Z_{o1} = Z_{o4} = Z_{o7} = 1$ and all the rest are zero in order to satisfy the constraint $X_{oj} \leq Z_{oj}M_{oj}$ for all arcs $(o,j)\varepsilon S$. The constraint $\sum_{(o,j)\varepsilon S} Z_{oj} \leq r$ is then satisfied and the solution is clearly feasible for PI.

IV. DISCUSSION

The problem, as presented, can also be solved using the partitioning procedure proposed by Benders (Ref. 3) or by enumerating all the possible combinations of r ports and selecting from these the combination that yields the minimum total cost in satisfying the total demand for imported goods. These methods are, however, not recommended when the number of ports in the system under consideration is fairly large while r is fairly small due to the amount of computational work required. In the case of the partitioning procedure, the computational burden can conceivably be great due to the size and number of integer programming problems that have to be solved. As for the enumeration method, the $\frac{(n-\#(K))!}{(r-\#(K))!(n-r)!}$ different combinations that have to be investigated would just be too numerous. As an example, consider the case of the Philippines which has 93 national ports with the port of Manila as the principal import port. A current problem is to determine other ports which can be developed to handle importations in order to relieve the port of Manila of the total burden. Assuming that two new import ports are desired (r=3), the enumeration method requires the investigation of 4186 different combinations. So, for large networks, the algorithm proposed presents a faster method of obtaining an acceptable solution. It requires at most n-r iterations and the Out of Kilter Algorithm is used only once. The amount of computations are increased only by the number of minimum route problems that have to be solved at each iteration. The total amount of computational work is still considerably less than that required by either the partitioning procedure or the enumeration method.

The solution algorithm presented divides the set of ports at each iteration until a feasible subset of r ports is obtained. The division is done by eliminating from further consideration at each iteration, that port whose exclusion contributes the least increase in total cost in satisfying the demands for imported goods when the demand at that particular port has to be satisfied through other ports in the set currently under consideration. By doing this, the algorithm will then disregard all the feasible sets whose elements include at least one of the ports eliminated. The algorithm does not necessarily yield an optimal solution since there can exist a better solution involving a port previously eliminated. This appears to be the main drawback of the algorithm.

Consider the example problem in Section IIIB. It can be shown that the solution for this problem given by the algorithm is optimal in the cases where r is greater than or equal to three. Suppose, instead, that two ports of entry are desired for the system. The solution from the algorithm is shown in Figure 5. The ports chosen as import ports are those corresponding to nodes 1 and 7 at a total cost of 2390. This, however, is not optimal since choosing the ports corresponding to the nodes 1 and 2 results in the least total cost. The minimum cost flow allocation in this case is shown in Figure 6 with a total cost of 2270. Nevertheless, even in this case, the solution is still a good solution since it is better than all the remaining possible choices.

In the event that an optimal solution is required, the proposed algorithm can be used to generate a good feasible solution, which is probably not too far from optimal, to start other algorithms yielding optimal solutions such as the partitioning algorithm of Benders (Ref. 3). It is felt that a substantial amount of computational work can be saved if this is done.

The proposed algorithm was developed under the assumption that development cost is the same for any port selected. This is more often not true because of the different sizes and characteristics of each port. If the assumption is relaxed, the number of ports of entry that can be created with the allocated special budget can no longer be determined beforehand and it becomes part of the problem. The constraint

$$\sum_{(o,j)\varepsilon S} Z_{oj} \leq r \text{ is replaced by } \sum_{(o,j)\varepsilon S} d_{oj} Z_{oj} \leq D.$$

The algorithm can be modified slightly to solve the resulting problem by obtaining at the start of each iteration the total development cost

$$\sum_{(o,j)\varepsilon J_s} d_{oj} Z_{oj} \text{ where } Z_{oj} = \begin{cases} 1 & \text{if } (o,j)\varepsilon J_s. \\ 0 & \text{otherwise.} \end{cases}$$

and comparing this with the special budget D. If the total development cost is less than or equal to D, then the problem is solved and the elements of the set J_s are selected. If, on the other hand, it is greater than D, the subroutine is used to select the arc (o,j*) to be eliminated. Upon eliminating the arc (o,j*), if the total development cost still remains larger than D, a new iteration is started. Otherwise, the algorithm terminates.

An extension of the problem results when it is decided beforehand that the system being considered should have exactly r ports of entry. In this case, the special budget constraint is replaced by the constraint

$$\sum_{(o,j)\varepsilon S} Z_{oj} = r \quad \text{where} \quad Z_{oj} = \begin{cases} 1 & \text{if } X_{oj} > 0. \\ 0 & \text{otherwise.} \end{cases}$$

After solving PII, the unconstrained solution obtained can be checked to determine if this new constraint is satisfied. If,

1. $\sum_{(o,j)\varepsilon S} Z_{oj} > r$, then $\#(J_1) > r$ and the algorithm, as presented, can be used to obtain a solution.

2. $\sum_{(o,j)\varepsilon S} Z_{oj} < r$, then $\#(J_1) < r$ which implies that the number of ports of entry which can attain the least total cost is less than the number required. The algorithm can be modified to solve the problem in this case. In determining which port $j\varepsilon\bar{J}_1$ can be added to the set J_1, the cost C_j of recirculating the demand M_{jn} through the arc (o,j) for all $j\varepsilon\bar{J}_1$ should be calculated. Then the port j^* for which

$$C_{j^*} = \underset{j\varepsilon\bar{J}_1}{\text{Minimum}}\ C_j$$

can be added to the set J_1.

3. $\sum_{(o,j)\varepsilon S} Z_{oj} = r$, the unconstrained solution is also optimum for the problem and the elements of the set J_1 are the recommended ports.

A final extension of the problem results when the assumption of linearity of costs is relaxed. It is reasonable to expect that a port of entry incurs some cost even when no goods are entered through it. This represents the fixed minimum cost necessary to maintain the personnel and facilities of the port. Also, the rate of increase in cost can be expected to decrease from an initially large value. However, since the quantity of goods entered at a port of entry is normally large, it is not too erroneous to assume that the rate of increase is constant. Hence, the operating and maintenance cost of a port can be expressed as the sum of a fixed cost and a linear multiple of the quantity of goods entered at that port.

The addition of a fixed cost changes the problem PI to the problem P′I as follows:

$$\text{Minimize} \quad \sum_{(o,j)\varepsilon S} c_{oj}X_{oj} + \sum_{(o,j)\varepsilon S} b_{oj}Z_{oj} + \sum_{(i,j)\varepsilon A} c_{ij}X_{ij} + \sum_{(i,n)\varepsilon T} c_{in}X_{in}$$

Subject to

$$\sum_{(o,j)\varepsilon S} X_{oj} = Q$$

$$\sum_{(i,k)\varepsilon A'} X_{ik} - \sum_{(j,i)\varepsilon A'} X_{ji} = 0 \text{ for all } i=1,\ldots,n-1$$

$$\sum_{(i,n)\varepsilon T} X_{in} = -Q$$

$$(\text{P}'\text{I}) \qquad \sum_{(o,j)\varepsilon S} Z_{oj} \leq r$$

$$L_{ij} \leq X_{ij} \leq M_{ij} \quad \text{for all } (i,j)\varepsilon A'$$

$$X_{oj} \leq Z_{oj}M_{oj} \quad \text{for all } (o,j)\varepsilon S$$

$$0 \leq Z_{oj} \leq 1 \quad Z_{oj} \text{ integer}$$

where b_{oj} is the fixed cost associated with the use of a port of entry.

Problem P′I reduces to a fixed cost transshipment problem P′II when the restriction on the number of source arcs used is removed. Malek-Zavarei and Frisch (Ref. 4) showed that a fixed cost transshipment problem can be converted to a fixed cost transportation problem for which some good approximate solution procedures are presently available (Refs. 5 and 6). So, it is conceivable that an algorithm similar to the one proposed in this report can be developed to solve P′I.

V. CONCLUSIONS

A. SUMMARY

The problem of selecting ports of entry from a network of ports linked together by commercial shipping routes was developed and an algorithm to solve the problem was presented. The algorithm initially considers all the ports as ports of entry and gradually reduces the number to that number desired for the system in such a way that the increase in cost is minimized.

The algorithm presented was not intended to be the only tool on which to base decisions. Rather, it was meant to provide the initial ground-work for the development of better solution procedures and, to provide some initial solutions to work with. The results of the algorithm were found not to be optimal in some cases but are considered good starting points from which to arrive at better solutions. The algorithm, however, is useful in the sense that it substantially reduces the work necessary to determine a good feasible solution especially in the case where the number of possible ports of entry is large enough to make the enumeration of all possible combinations of the ports impractical and in the cases where solutions not far from optimal are deemed sufficient.

B. RECOMMENDATIONS

The author believes that an optimal solution to the problem can be attained without resorting to the enumeration method and that the solution process presented can be developed further to yield an optimal solution in all cases. An attempt should be made at determining the areas where the algorithm fails prior to extending the algorithm.

Finally, the extension of the problem resulting from the addition of fixed costs should be an interesting subject for future studies. It is felt that a solution process similar to the one proposed in this thesis can be developed for that case.

APPENDIX A

FIGURES

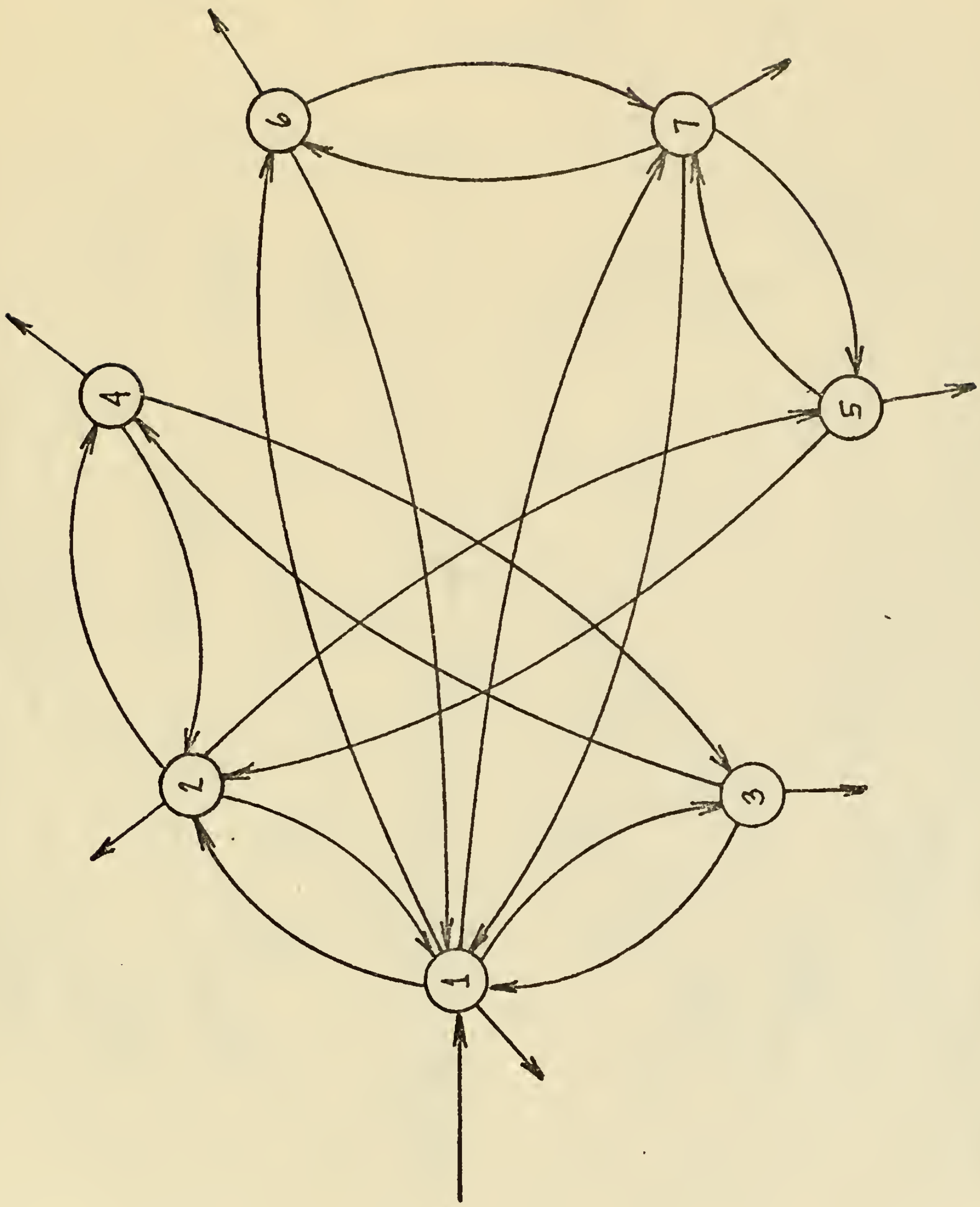

Figure 1

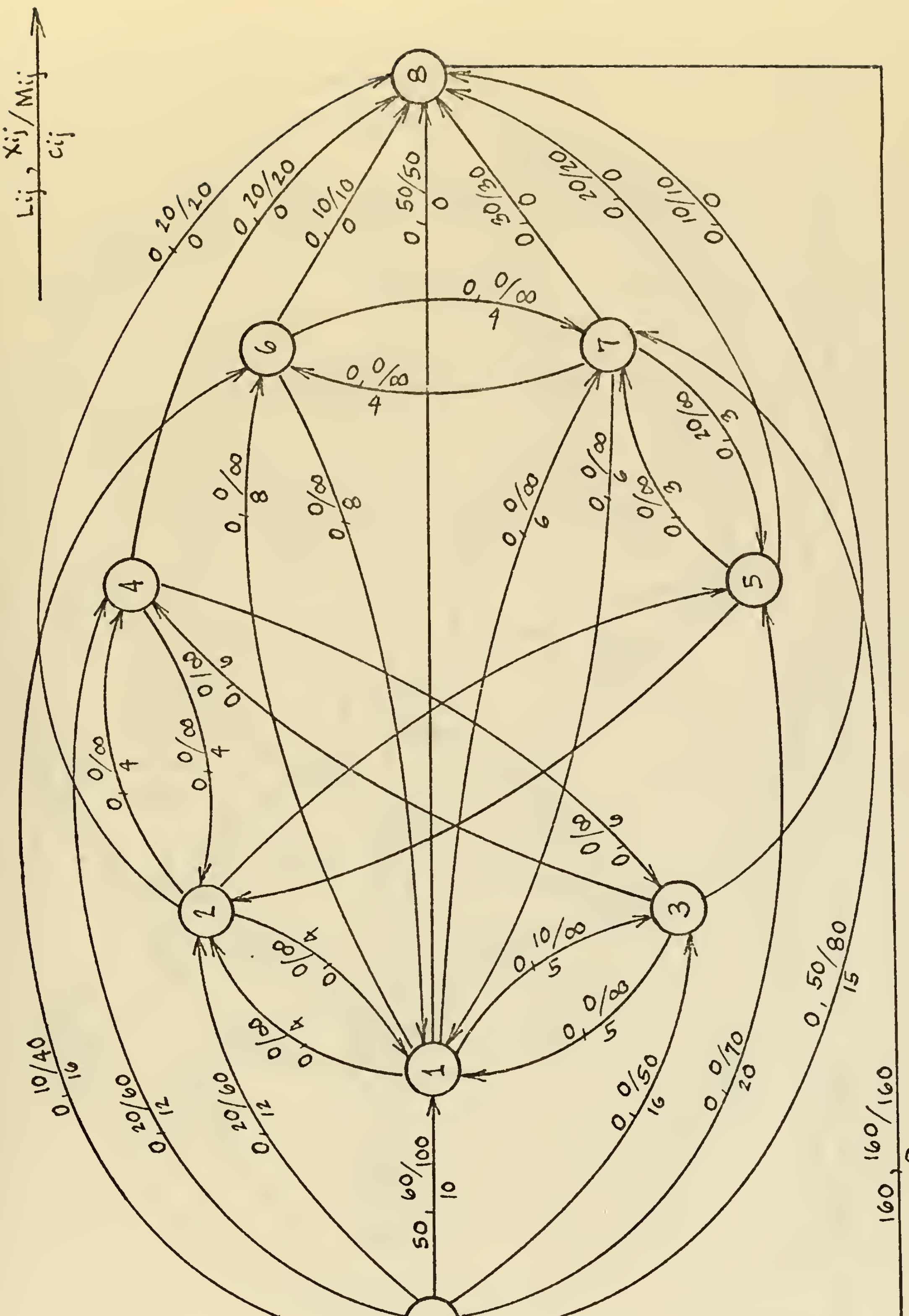

Figure 2

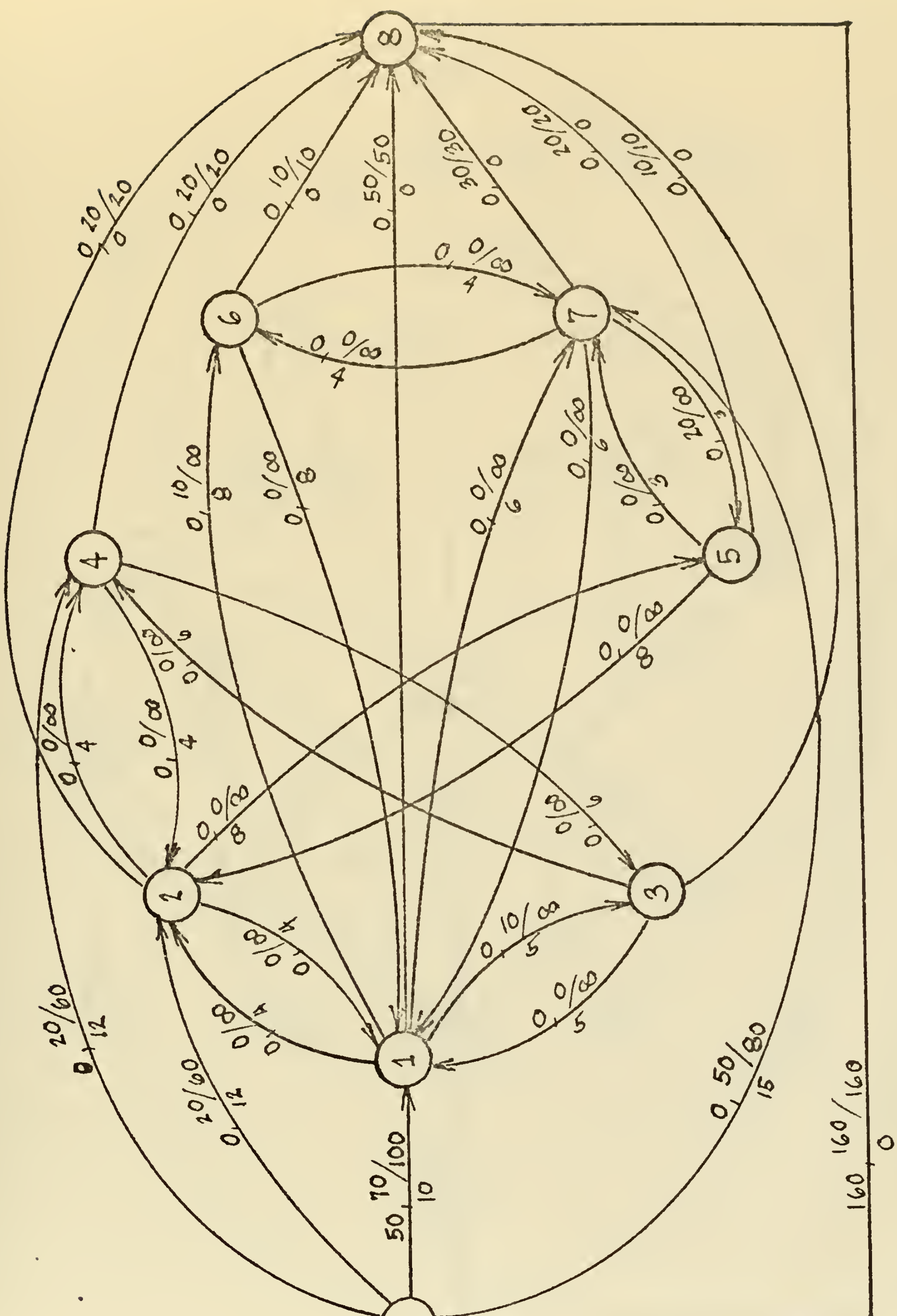

Figure 3

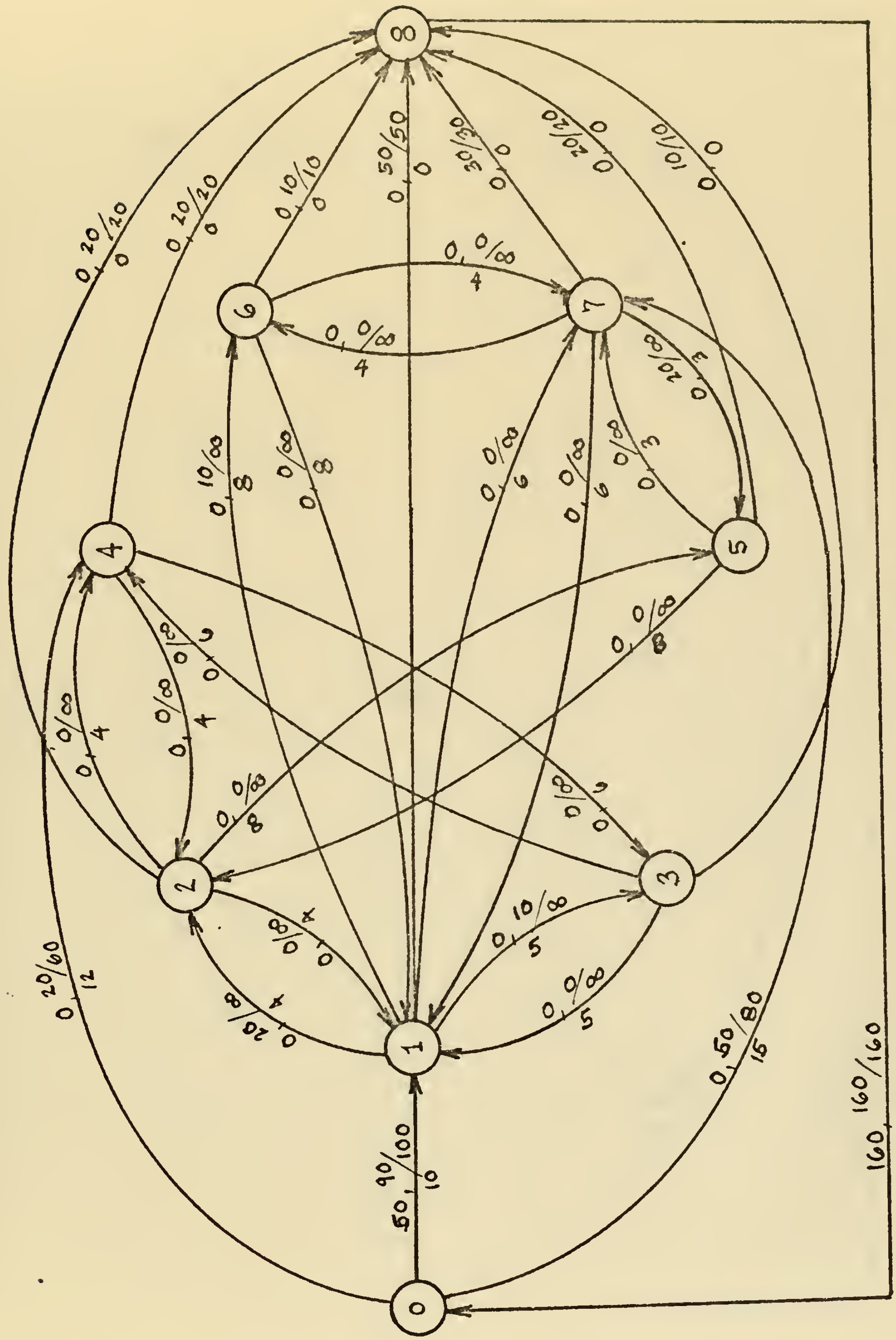

Figure 4

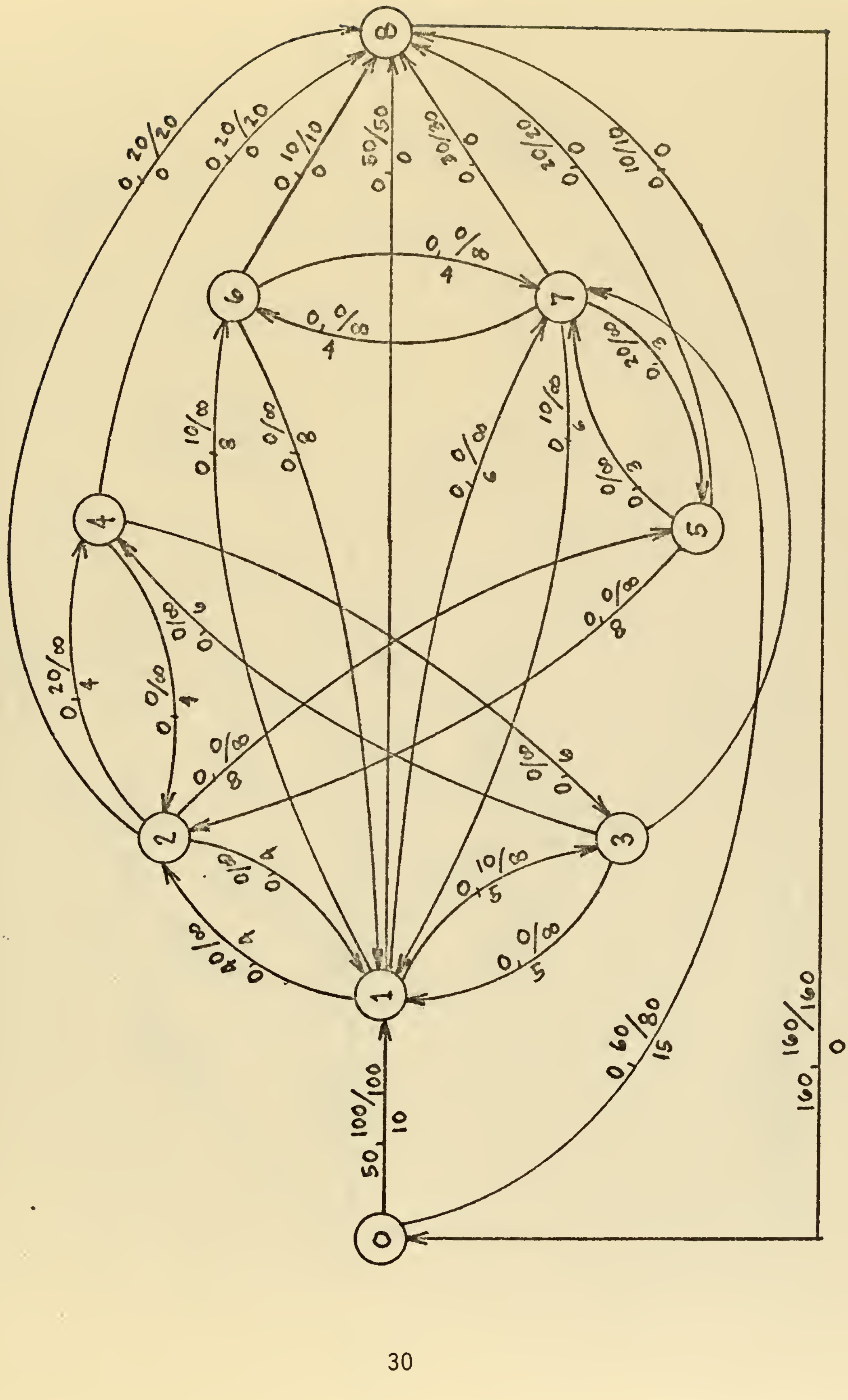

Figure 5

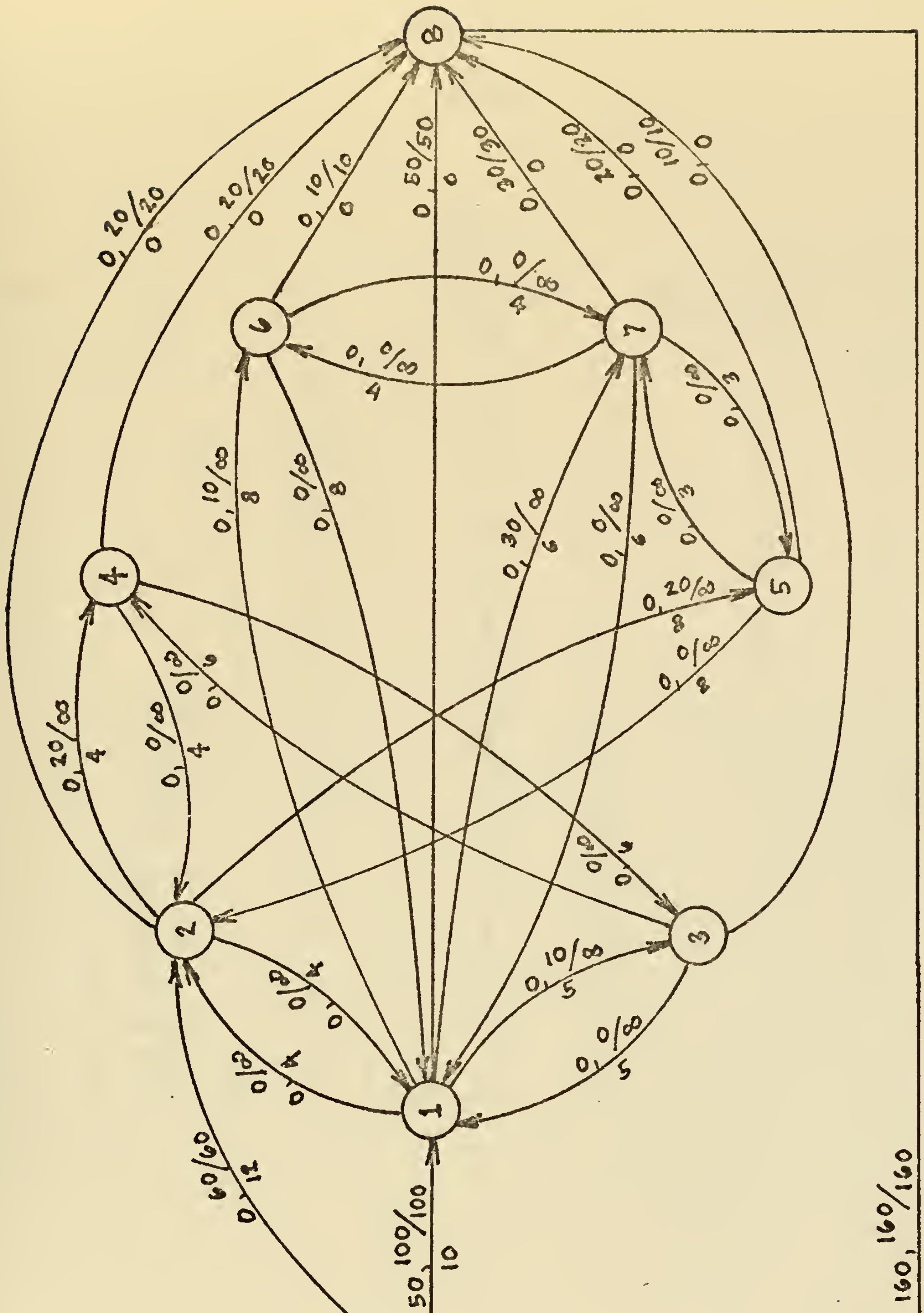

Figure 6

APPENDIX B

TABLES

Table 1

PORT i	LEG (i,j)	COST c_{ij}	LOWER BOUND L_{ij}	UPPER BOUND ON CAPACITY M_{ij}	DEMANDS M_{in}
1		10	50	100	50
2		12	0	60	20
3		16	0	50	10
4		12	0	60	20
5		20	0	70	20
6		16	0	40	10
7		15	0	80	30
	(1,2)	4	0		
	(1,3)	5	0		
	(1,6)	8	0		
	(1,7)	6	0		
	(2,4)	4	0		
	(2,5)	8	0		
	(3,4)	6	0		
	(5,7)	3	0		
	(6,7)	4	0		

Table 2

ARC	ROUTE $R_{oj}^{(m)}$	$a_{oj}^{(m)}$	$MIN(M_{oj}-X_{oj})$	C_j
(0,2)	0 - 1 - 2	2	40	40
(0,4)	0 - 2 - 4	4	40	80
(0,6)	0 - 1 - 6	2	40	20 *
(0,7)	0 - 1 - 7	1	40	40
	0 - 2 - 3 - 7	2	20	60

Table 3

ARC	ROUTE $R_{oj}^{(m)}$	$a_{oj}^{(m)}$	$MIN(M_{oj}-X_{oj})$	C_j
(0,2)	0 - 1 - 2	2	40	40 *
(0,4)	0 - 2 - 4	4	40	80
(0,7)	0 - 1 - 7	1	30	30
	0 - 2 - 5 - 7	2	20	70

APPENDIX C

EXAMPLE FORMULATION

PI: Minimize

$$10X_{01} + 12X_{02} + 16X_{03} + 12X_{04} + 20X_{05} + 16X_{06} + 15X_{07} +$$
$$4(X_{12} + X_{21}) + 5(X_{13} + X_{31}) + 8(X_{16} + X_{61}) + 6(X_{17} + X_{71}) +$$
$$4(X_{24} + X_{42}) + 8(X_{25} + X_{52}) + 6(X_{34} + X_{43}) + 3(X_{57} + X_{75}) +$$
$$4(X_{67} + X_{76}) + 0(X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78})$$

Subject to

$$X_{01} + X_{02} + X_{03} + X_{04} + X_{05} + X_{06} + X_{07} = 160$$
$$X_{18} + X_{12} + X_{13} + X_{16} + X_{17} - X_{01} - X_{21} - X_{31} - X_{61} - X_{71} = 0$$
$$X_{28} + X_{21} + X_{24} + X_{25} - X_{12} - X_{42} - X_{52} - X_{02} = 0$$
$$X_{38} + X_{31} + X_{34} - X_{13} - X_{43} - X_{03} = 0$$
$$X_{48} + X_{42} + X_{43} - X_{24} - X_{34} - X_{04} = 0$$
$$X_{58} + X_{52} + X_{57} - X_{25} - X_{75} - X_{05} = 0$$
$$X_{68} + X_{61} + X_{67} - X_{16} - X_{76} - X_{06} = 0$$
$$X_{78} + X_{71} + X_{75} + X_{76} - X_{17} - X_{57} - X_{67} - X_{07} = 0$$
$$X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78} = -160$$
$$Z_{01} + Z_{02} + Z_{03} + Z_{04} + Z_{05} + Z_{06} + Z_{07} \le 3$$

$50 \le X_{01} \le 100Z_{01}$ $\quad 0 \le X_{18} \le 50$ $\quad 0 \le X_{ij} \le \infty$

$0 \le X_{02} \le 60Z_{02}$ $\quad 0 \le X_{28} \le 20$ $\quad$ for all $i,j=1,\dots,7$

$0 \le X_{03} \le 50Z_{03}$ $\quad 0 \le X_{38} \le 10$ $\quad 0 \le Z_{oj} \le 1$, Z_{oj} integer

$0 \le X_{04} \le 60Z_{04}$ $\quad 0 \le X_{48} \le 20$ $\quad$ for all $j=1,\dots,7$

$0 \le X_{05} \le 70Z_{05}$ $\quad 0 \le X_{58} \le 20$

$0 \le X_{06} \le 40Z_{06}$ $\quad 0 \le X_{68} \le 10$

$0 \le X_{07} \le 80Z_{07}$ $\quad 0 \le X_{78} \le 30$

PII: Minimize $10X_{01} + 12X_{02} + 16X_{03} + 12X_{04} + 20X_{05} + 16X_{06} + 15X_{07} +$

$4(X_{12} + X_{21}) + 5(X_{13} + X_{31}) + 8(X_{16} + X_{61}) + 6(X_{17} + X_{71}) +$

$4(X_{24} + X_{42}) + 8(X_{25} + X_{52}) + 6(X_{34} + X_{43}) + 3(X_{57} + X_{75}) +$

$4(X_{67} + X_{76}) + 0(X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78})$

Subject to

$$X_{01} + X_{02} + X_{03} + X_{04} + X_{05} + X_{06} + X_{07} = 160$$

$$X_{18} + X_{12} + X_{13} + X_{16} + X_{17} - X_{21} - X_{31} - X_{61} - X_{71} - X_{01} = 0$$

$$X_{28} + X_{21} + X_{24} + X_{25} - X_{12} - X_{42} - X_{52} - X_{02} = 0$$

$$X_{38} + X_{31} + X_{34} - X_{13} - X_{43} - X_{03} = 0$$

$$X_{48} + X_{42} + X_{43} - X_{24} - X_{34} - X_{04} = 0$$

$$X_{58} + X_{52} + X_{57} - X_{25} - X_{75} - X_{05} = 0$$

$$X_{68} + X_{61} + X_{67} - X_{16} - X_{76} - X_{06} = 0$$

$$X_{78} + X_{71} + X_{75} + X_{76} - X_{17} - X_{57} - X_{67} - X_{07} = 0$$

$$X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78} = -160$$

$50 \le X_{01} \le 100$	$0 \le X_{18} \le 50$	$0 \le Xij \le \infty$
$0 \le X_{02} \le 60$	$0 \le X_{28} \le 20$	for all i,j=1,...,7
$0 \le X_{03} \le 50$	$0 \le X_{38} \le 10$	
$0 \le X_{04} \le 60$	$0 \le X_{48} \le 20$	
$0 \le X_{05} \le 70$	$0 \le X_{58} \le 20$	
$0 \le X_{06} \le 40$	$0 \le X_{68} \le 10$	
$0 \le X_{07} \le 80$	$0 \le X_{78} \le 30$	

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A solution algorithm is presented which solves the problem of selecting ports of entry for the case where the number of ports of entry is constrained to be small relative to the total number of ports. The algorithm initially considers all ports in the system considered as candidates for ports of entry and proceeds by eliminating from further consideration one port at a time until the required number is attained. An

20. Abstract Cont

attempt is made to remain as close as possible to the unconstrained solution by eliminating at each iteration the port that has the least effect on the objective function value. It is pointed out that the algorithm can yield non-optimal results in some cases but the solution is still better than most feasible solutions.

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